Improvements of the local bosonic algorithm.

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We report on several improvements of the local bosonic algorithm proposed by M. Lüscher. We find that preconditioning and over-relaxation works very well. A detailed comparison between the bosonic and the Kramers-algorithms shows comparable performance for the physical situation examined.

1. INTRODUCTION

Since its proposal in [1], the local bosonic fermion method has been thoroughly investigated and improved [2–5,9–14]. We report here on some improvements, especially the efficient implementation of preconditioning techniques, and give more detailed information on the comparison of this algorithm to the Kramers algorithm presented in [15]. As in [2], all investigations were made with the gauge group $SU(2)$.

2. PRECONDITIONING

Even-odd preconditioning is known to work very well for iterative solvers. It is therefore straightforward to try to apply this technique to the local bosonic algorithm. We quickly recall that the hermitean dirac operator can be written as

$$Q = c_0\gamma_5 \begin{pmatrix} 1 & -KD_{eo} \\ -KD_{oe} & 1 \end{pmatrix}. \quad (1)$$

$c_0 = [c_M(1+8K)]^{-1}$ with $c_M > 1$ is a normalization constant so that $\|Q\| \le 1$. Using the following identity,

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det A \det\left(D - CA^{-1}B\right), \quad (2)$$

we can immediately write down the preconditioned matrix

$$\hat{Q} = \tilde{c}_0\gamma_5(1 - K^2 D_{oe}D_{eo}) \quad (3)$$

with $\tilde{c}_0 = [c_M(1+64K^2)]^{-1}$ and $\|\hat{Q}\| \le 1$. This matrix, however, contains next-to-nearest neighbour interactions. Since it appears squared in the local bosonic action, the local updates become rather complicated. The local bosonic approximation to the fermion determinant is

$$\begin{aligned} \det \hat{Q}^2 &\approx [\det P(\hat{Q}^2)]^{-1} \\ &= \prod_k [\det(\hat{Q}^2 - z_k)]^{-1} \\ &= \prod_k [\det(\hat{Q} - r_k)(\hat{Q} - r_k^*)]^{-1}, \end{aligned} \quad (4)$$

with P a suitably chosen approximation to $1/x$. By applying Equation (2) to each single factor in this equation, we get

$$\det\left(\hat{Q} - r_k\right) \propto \det \begin{pmatrix} \tilde{c}_0\gamma_5 & -\tilde{c}_0\gamma_5 KD_{eo} \\ -\tilde{c}_0\gamma_5 KD_{oe} & \tilde{c}_0\gamma_5 - r_k \end{pmatrix}. \quad (5)$$

Letting $\tilde{Q} = \tilde{c}_0/c_0 Q$, we obtain the preconditioned action

$$\tilde{S}_b = \sum_k \phi_k^\dagger (\tilde{Q} - P_o r_k^*)(\tilde{Q} - P_o r_k)\phi_k, \quad (6)$$

which is very similar to the original one. P_o denotes the projector on the odd sites.

The dynamics of the preconditioned algorithm is investigated in Table 1. As can be seen, preconditioning saves a factor of about 4 in computer time, as expected. The analysis of the condition numbers of Q^2 and $\hat{Q}^2$ show that the gain is even near 8. The reason for that is that the largest eigenvalue of $\hat{Q}^2$ is far below 1. The attempt to exploit this fact by adjusting c_M fails because when the largest eigenvalue of $\hat{Q}^2$ gets near 1 the bosonic fields develop slow modes. Detailed investigations show that there is a tradeoff between

n, ϵ	c_M	Updating	$\langle\Box\rangle$	$\langle\hat{\lambda}_{\min}\rangle$	$\langle\hat{\lambda}_{\max}\rangle$	$\tau_{\rm int}(\Box)$
0.0017	1.1	HOh,non-pre	0.4715(12)	0.00321(8)	0.6485(1)	225(62)
50		HOh	0.4743(24)	0.00614(18)	0.2617(1)	196(52)
0.0045		HOh	0.4707(15)	0.00624(16)		45(10)
30		HOoOo	0.4704(11)	0.00609(8)		15(4)
0.015	0.6	HOoOo	0.4710(16)	0.00612(11)	0.8797(3)	16(5)
16	0.58	HOoOo	0.4692(13)	0.00617(6)	0.9414(3)	23(5)

Table 1
Autocorrelation times in units of $1000Q\phi$-multiplications on the $4^3\times 8$ lattice for $\beta = 1.75$ and $K = 0.165$. n and ϵ are chosen so that $\delta = 0.03$. All except the first run are preconditioned. The letters in the third column give the type and order of sweeps used per iteration, where H is a bosonic heatbath, O a bosonic over-relaxation and h and o the gauge updates.

the slow bosonic modes and the number of fields. This is however not yet analytically understood.

The idea of applying some reverse transformation to the single factors of the polynomial can easily be applied to different cases of preconditioning, e.g. preconditioning with the inverse free fermion matrix, still resulting in a local algorithm.

3. HYBRID OVER-RELAXATION

It is believed that the application of local heatbath and over-relaxation sweeps in a combination $1 : \xi$, where ξ is a suitably defined correlation length, results in a dynamical critical exponent of $z \approx 1$ (see e.g. [7]). It was found in [3] that the application of this method to the gauge fields or the bosonic fields only has no effect, which can be understood as the fact that the coupling of the fields dominates the dynamics of the algorithm in the examined case. The application to both fields however is very efficient, as can be seen from Table 2. Although this is far from a verification of $z \approx 1$, we believe that the gain will increase with increasing correlation length.

4. COMBINED UPDATE

In [3] an update which updates the bosonic and gauge fields at the same time was proposed. The method uses the fact that we can update a gauge link $U(x, \mu)$ according to the effective action after integrating out all bosonic degrees of freedom

Updating	$\langle\Box\rangle$	$\tau_{\rm int}(\Box)$
HOh	0.4707(15)	45(10)
HOo	0.4701(12)	30(8)
HoOo	0.4708(14)	20(4)
HOoOo	0.4704(11)	15(4)
HoOoOo	0.4702(8)	19(5)

Table 2
Autocorrelation times in units of $1000Q\phi$-multiplications. The run parameters are the same as in the third line of Table 1.

$\phi(x)$ and $\phi(x + \hat{\mu})$ and then refresh $\phi(x)$ and $\phi(x+\hat{\mu})$ with heatbath-updates. The hope is, that the effective action induces a smaller fermionic force on the gauge field, which was found to dominate the update. Indeed, it is only of order K^2 instead of K in the standard update; however the updating procedure is more complicated what may compensate for this effect. There also exists no free-field analysis to get a hint about the dynamical critical exponent of this update. Table 3 shows that the update performs comparably to the standard updates. However its implementation is much more difficult and its application to improved actions is probably not feasible.

5. ALGORITHM COMPARISON

In [15] the comparison between the local bosonic algorithm in its hermitean version and

n, ϵ	c_M	Upd	$\langle\Box\rangle$	$\tau_{\rm int}(\Box)$
0.015	0.6	a	0.4708(15)	20(5)
16		b	0.4720(14)	17(4)

Table 3
Autocorrelation times for the combined update. The first line uses one combined over-relaxation step, for the second a standard bosonic over-relaxation step was added.

the Kramers algorithm has been described. We will give more detailed information about the runs made and the parameters chosen. The bosonic parameters were chosen according to the investigations presented above while the Kramers algorithm was examined in [6]. In Table 4 the parameters for both algorithms are given as well as the expectation values of the extremal eigenvalues of $\hat{Q}^2$ (with c_M given as in the Boson section of the table). In Table 5 the results for the observables are presented. As can be seen, the parameters were chosen in a way that the observables agree; it seems however that δ is slightly too big in the $8^3 12$ case.

	$6^3 12$	$8^3 12$	16^4
Machine	Q1	Q1	QH2
$\langle\lambda_{\rm min}\rangle$	0.0115(4)	0.00539(9)	0.00478(3)
$\langle\lambda_{\rm max}\rangle$	0.9386(4)	0.6100(2)	0.6961(5)
Kramers			
ϵ_{md}	0.205	0.185	0.125
k	3	4	5
γ	0.5	0.5	0.5
Boson			
ϵ	0.01454	0.0061	0.0048
n	18	24	44
δ	2%	4%	0.38%
c_M	0.6	0.745	0.7

Table 4
Technical parameters and eigenvalues of $\hat{Q}^2$ for both algorithms.

	Alg	$6^3 12$	$8^3 12$	16^4
$\langle\Box\rangle$	K	0.5803(2)	0.5777(3)	0.5778(1)
	B	0.5804(4)	0.5768(2)	0.5779(1)
m_π	K	1.191(8)	1.052(8)	1.003(2)
	B	1.170(12)	1.044(3)	1.004(4)
m_ρ	K	1.275(9)	1.123(11)	1.060(2)
	B	1.254(14)	1.112(4)	1.063(5)
$\tau(\Box)$	K	480(100)	979(300)	540(230)
[sec]	B	289(20)	1781(112)	990(330)

Table 5
Results for both algorithms.

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